1. Use the conservation of material equation to model a one-dimensional concentration diffusion using the data below:

$$\frac{\partial C(x,t)}{\partial t} = D\frac{\partial^2 C(x,t)}{\partial x^2}.$$

- **Diffusivity:** $D = 2\ \mathrm{m}^2/\mathrm{s}$
- **Time step:** $\Delta t = 0.1\ \mathrm{s}$
- **Total time:** $t_{\mathrm{total}} = 100\ \mathrm{s}$
- **Space step:** $\Delta x = 1\ \mathrm{m}$
- **Initial concentration profile and boundary conditions:**

$$C(x,0) = [1,2,3,4,\dots\ 32]\ \mathrm{mol/m^3} \quad \text{and} \quad \left.\frac{\partial C}{\partial x}\right|_{x=0} = \left.\frac{\partial C}{\partial x}\right|_{x=32} = 0$$

- **Condition:** No internal production or consumption of material

Use finite Difference methode (Central-Difference Rule) for solving this PDE. A colored diagram representation of species diffusion must be provided respect to time and space. Use **MATLAB** for this problem.
Extra-Credit: Using Keller-Box method

2. Most high-energy cells use lithium metal for the negative electrode; furthermore, rechargeable lithium systems rely on intercalation for reversible reactions at the cathode. Discuss the idea of replacing Li with Mg for future rechargeable cells. Specifically, contrast and compare Li and Mg by commenting on the following:

   (a) **Specific capacity** $[\frac{\mathrm{Ah}}{\mathrm{g}}, \text{metal}]$

   (b) **Volumetric capacity** $[\frac{\mathrm{Ah}}{\mathrm{cm}^3}, \text{metal}]$

   (c) **Earth abundance**

   (d) **Specific energy and energy density**

   (e) **Ionic radii**

   (f) **Charge/radius ratio**

3. Lithium-ion batteries have self-discharge rates of 1-2% per month. If two adjacent cells in a long string connected in series have rates of self-discharge of 1 and 2% per month, respectively, and the battery is fully charged each month, how long before the SOCs of these two cells vary by 5%? The rate of self-discharge, however, can be as high as 5% in the first 24 hours. If the initial rates of self-discharge for the two cells are 3 and 5%, respectively, how does the answer change? What role would the battery management system play in this scenario?