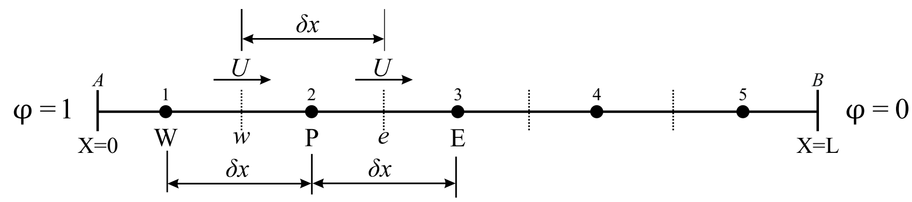
Homework #9	**Computational Fluid Dynamics I**	Prof. V. Esfahanian
Due Date: 3/11/97		TA: A.Bahrami & M.Fazli

1. For steady one-dimensional convection-diffusion problem, i.e.

$$\frac{d}{dx}(\rho u \phi) = \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right)$$

by using upwind control volume method for the problem shown below



obtain the following table and show all the details.

Node	a_W	a_E	S_P	S_u
1	0	D	$-(2D+F)$	$(2D+F)\phi_A$
2, 3, 4	$D+F$	D	0	0
5	$D+F$	0	$-2D$	$2D\phi_B$

2. The governing equation for steady convection-diffusion in one-dimensional is

$$\frac{d}{dx}(\rho u \phi) = \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right)$$

Show that the exact solution to the above equation with the boundary conditions given below:

$$\begin{cases} x = 0 & \phi = \phi_P \\ x = \delta x & \phi = \phi_E \end{cases}$$

is

$$\frac{\phi - \phi_P}{\phi_E - \phi_P} = \frac{e^{\mathrm{Pe}\cdot x/\delta x} - 1}{e^{\mathrm{Pe}} - 1}, \qquad \mathrm{Pe} = \frac{(\rho u)\delta x}{\Gamma}$$

(a) Plot the above solution for Pe=-1, 0, 1 and for very large and small Peclet number.

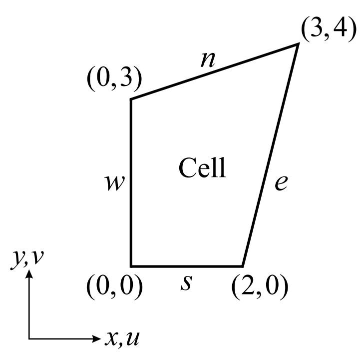
(b) Let us use the exact solution to compute ϕ_e and ϕ_w in the difference equation. Show that the coefficients for the discrete equation will be

$$
\begin{aligned}
a_E &= \frac{F_e}{e^{\mathrm{Pe}_e} - 1} \\
a_W &= \frac{F_w e^{\mathrm{Pe}_w}}{e^{\mathrm{Pe}_w} - 1} \\
a_P &= a_E + a_W + (F_e - F_w)
\end{aligned}
$$

The discretization done following the exact solution is known as exponential scheme. Despite its accuracy and highly desirable behavior, it is not very widely used because exponentials are expensive to compute.

3. The following Table and Figure represent the values of velocity components and ϕ scalar amount on the volume faces. Assuming the constant density:

- Calculate the mass flux (based on unit depth) at n, s, and w faces.
- Using the continuity equation, calculate the u velocity amount at e face.
- By ignoring the source and diffusion terms, obtain the ϕ value on the right face.



Face	velocity u	velocity v	Scalar φ
e	?	0	?
n	6	3	4
w	5	2	2
s	5	2	3.5