Quiz #5 - Takehome **Computational Fluid Dynamics I** Prof. V. Esfahanian
Due date: 29/8/97 TA: A. Bahrami & M. Fazli

1. Consider the following eigenvalue problem:

$$-\nu''(x) = \gamma\nu(x), \quad 0 < x < 1$$
$$\nu'(0) = 0 = \nu'(1)$$

(a) Show that employing one-sided finite difference scheme at the boundaries (backward and forward) as well as use of following discretization in the uniform grid $h = 1/N, \quad i = 0, 1, 2, ..., N, \quad x_i = -h/2 + ih$ result in a matrix form equation depicted below:

$$-V_{n-1} + 2V_n - V_{n+1} = \lambda V_n, \quad n = 1, 2, ...N$$
$$V_1 - V_0 = 0 = V_{N+1} - V_N$$

$$\begin{bmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & & & & \\ & \ddots & \ddots & \ddots & \\ & & & & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ \vdots \\ V_{N-1} \\ V_N \end{bmatrix} = \lambda \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ \vdots \\ V_{N-1} \\ V_N \end{bmatrix} \qquad \mathbf{A}\vec{V} = \lambda\vec{V}$$

(b) Prove that the eigenvales of matrix A are as follow:

$$\lambda_n = 2 - 2\cos\frac{(n-1)\pi}{N}, \quad n = 1, 2, .., N$$

and the corresponding eigenvectors are:

(c) without any mathematical operations prove that $V_n.V_m = 0$ for $n \neq m$ (hint: $A = A^T$)

(d) show that $V_1.V_1 = N, V_n.V_n = N/2$ for $n = 2, 3, .., N$

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V_n = \begin{bmatrix} \cos(n-1)\pi x_1 \\ \cos(n-1)\pi x_2 \\ \vdots \\ \vdots \\ \\ \cos(n-1)\pi x_N \end{bmatrix} \qquad n = 1, 2, \ldots, N
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