1. Derive a third-order finite-difference approximation to a first derivative in the form
$$\left(\frac{\partial u}{\partial x}\right)_i = \frac{au_{i-2} + bu_{i-1} + cu_i + du_{i+1}}{\Delta x}$$
Find the leading error term.

2. Derive a finite-difference approximation to a third derivative in the form
$$\left(\frac{\partial^3 u}{\partial x^3}\right)_i = \frac{au_{i-2} + bu_{i-1} + cu_i + du_{i+1} + eu_{i+2}}{(\Delta x)^3}$$
Find the leading error term.

3. Derive a finite-difference approximation to a first derivative in the form
$$a\left(\frac{\partial u}{\partial x}\right)_{i-1} + \left(\frac{\partial u}{\partial x}\right)_i = \frac{bu_{i-1} + cu_i + du_{i+1}}{\Delta x}$$
Find the leading error term.

4. Establish the following relations:

   a) $\nabla = E^{-1}\Delta$

   b) $\Delta\nabla = \nabla\Delta = \Delta - \nabla = \delta^2$

   c) $\mu\delta = \frac{1}{2}(\Delta + \nabla)$

   d) $\mu^2 = 1 + \frac{1}{4}\delta^2$

5. Show that the 2^{nd} derivative becomes
$$h^2 y_i'' = \left(\Delta^2 - \frac{1}{12}\Delta^4 + \frac{1}{12}\Delta^6 - \cdots\right) y_{i-1}$$
using finite-difference operator theory.

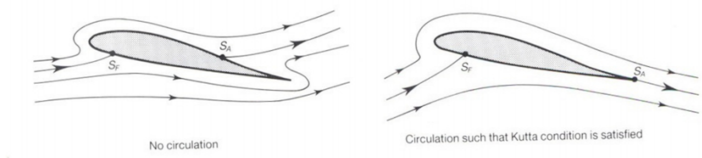

Figure 1: Kutta condition over an airfoil

6. By refer to section 3-8 of the Book, determine the order of averaging operator (μ).

7. Describe how to find the value of the stream function at the airfoil surface, ψ_0, to satisfy Kutta condition neumerically (Figure 1).

   Hint: Review the flow over a cylinder from Fluid Mechanics I (potential flow chapter).